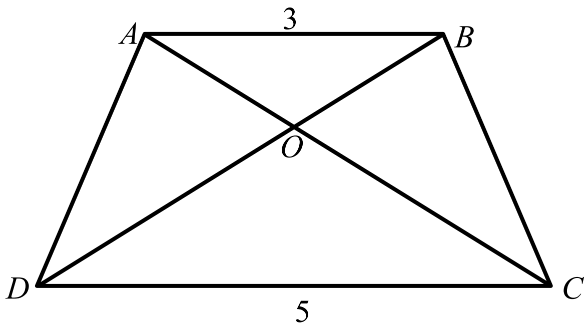
Hong Kong Mathematics Olympiad (2012 / 2013)
Heat Event (Group)
香港數學競賽 (2012 / 2013)
初賽項目(團體)

除非特別聲明，答案須用數字表達，並化至最簡。
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

1. 已知一個直角三角形三邊的長度皆為整數，且其中兩邊的長度為方程 $x^2-(m+2)x+4m=0$ 的根。求第三邊長度的最大值。
Given that the length of the three sides of a right-angled triangle are integers, and two of them are the roots of the equation $x^2-(m+2)x+4m=0$. Find the maximum length of the third side of the triangle.

2. 圖一所示為一梯形 $ABCD$，其中 $AB=3$、$CD=5$ 及對角線 AC、BD 相交於點 O。若 ΔAOB 的面積是 27，求梯形 $ABCD$ 的面積。
Figure 1 shows a trapezium $ABCD$, where $AB=3$, $CD=5$ and the diagonals AC and BD meet at O. If the area of ΔAOB is 27, find the area of the trapezium $ABCD$.

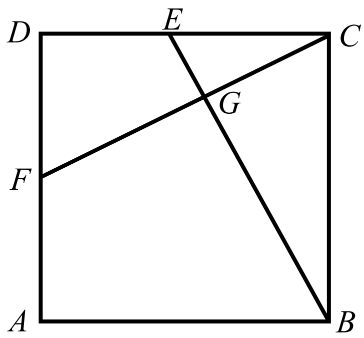


圖一
Figure 1

3. 設 x 及 y 為實數使得 $x^2+xy+y^2=2013$。求 x^2-xy+y^2 的最大值。
Let x and y be real numbers such that $x^2+xy+y^2=2013$. Find the maximum value of x^2-xy+y^2.

4. 若 α、β 是方程 $x^2+2013x+5=0$ 的根，求 $\left(\alpha^2+2011\alpha+3\right)\left(\beta^2+2015\beta+7\right)$ 的值。
If α, β are roots of $x^2+2013x+5=0$, find the value of $\left(\alpha^2+2011\alpha+3\right)\left(\beta^2+2015\beta+7\right)$.

5. 如圖二所示，$ABCD$ 為一個邊長為 10 單位的正方形，E 及 F 分別為 CD 及 AD 的中點，BE 及 FC 相交於 G。求 AG 的長度。
As shown in Figure 2, $ABCD$ is a square of side 10 units, E and F are the mid-points of CD and AD respectively, BE and FC intersect at G. Find the length of AG.



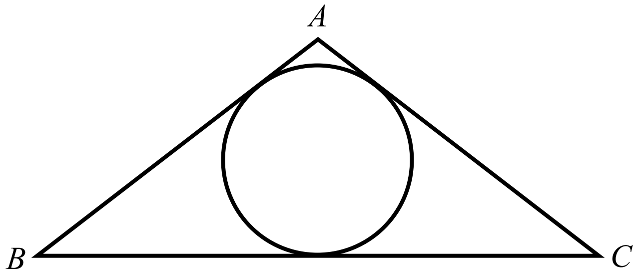
圖二

Figure 2

6. 若 a 及 b 為正實數，且方程 $x^2+ax+2b=0$ 及 $x^2+2bx+a=0$ 都有實數根。求 $a+b$ 的最小值。
If a and b are positive real numbers, and the equations $x^2+ax+2b=0$ and $x^2+2bx+a=0$ have real roots. Find the minimum value of $a+b$.

7. 已知 ΔABC 的三邊的長度組成一個等差數列，且為方程 $x^3-12x^2+47x-60=0$ 的根，求 ΔABC 的面積。
Given that the length of the three sides of ΔABC form an arithmetic sequence, and are the roots of the equation $x^3-12x^2+47x-60=0$, find the area of ΔABC.

8. 圖三中，ΔABC 為一等腰三角形，其中 $AB=AC$, $BC=240$。已知 ΔABC 的內接圓的半徑是 24，求 AB 的長度。
In Figure 3, ΔABC is an isosceles triangle with $AB=AC$, $BC=240$. The radius of the inscribed circle of ΔABC is 24. Find the length of AB.



圖三
Figure 3

9. 從 1 , 2 , 3 , … , 2012 , 2013 中最多可取出多少個數，使得在取出的數中任意兩個數之和都不是這兩個數之差的倍數？

At most how many numbers can be taken from the set of integers : 1 , 2 , 3 , … , 2012 , 2013 such that the sum of any two numbers taken out from the set is not a multiple of the difference between the two numbers.

10. 對所有正整數 n ，定義函數 f 為

(i) $f(1)=2012$ ，

(ii) $f(1)+f(2)+\cdots+f(n-1)+f(n)=n^2 f(n)$ ， $n>1$

求 $f(2012)$ 的值。

For all positive integers n , define a function f as

(i) $f(1)=2012$,

(ii) $f(1)+f(2)+\cdots+f(n-1)+f(n)=n^2 f(n)$, $n>1$

Find the value of $f(2012)$.

完

END